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Perturbation Technique and Differential Transform Method for Mixed Convection Flow on Heat and Mass Transfer with Chemical Reaction

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ABSTRACT

A new analytical solution is introduced for the effect of chemical reaction on mixed convective heat and mass transfer in a vertical double passage channel. The vertical channel is divided into two passages (by means of a baffle) for two separate flow streams. Each stream has its own individual velocity, temperature and concentration fields. After placing the baffle the fluid is concentrated in one of the passage. Approximate analytical solutions are found for the coupled nonlinear ordinary differential equations using regular perturbation method (PM) and Differential Transform method (DTM). The validity of the Differential Transform series solutions are verified with the regular perturbation method. The velocity, temperature and concentration solutions are obtained and discussed for various physical parameters such as thermal Grashoff number, mass Grashoff number, Brinkman number and chemical reaction parameter at different positions of the baffle. It is found that the thermal Grashoff number, mass Grashoff number, Brinkman number enhances the flow whereas chemical reaction parameter reduces the flow at all baffle positions. It is also found that as Brinkman number increases the DTM and PM show more error.

KEYWORDS: *Baffle, first order chemical reaction, mixed convection, perturbation method, Differential Transform method.*

I. INTRODUCTION

Study of mixed convection in the channel has been to the focus of lot of investigation during the last three decades because of the multiple applications in which it is involved. These includes cooling of electronic equipment, heat exchangers, chemical processing equipment, gas-cooled nuclear reactors and others. Tao [1] analyzed the laminar fully developed mixed convection flow in a vertical parallel-plate channel with uniform wall temperatures. Aung and Worku [2, 3] discussed the theory of combined free and forced convection in a vertical channel with flow reversal conditions for both developing and fully developed flows. The case of developing mixed convection flow in ducts with asymmetric wall heat fluxes was analyzed by the same authors [4]. Recently, Prathap Kumar et al. [5] and Umavathi et al. [6, 7] studied the mixed convective flow and heat transfer in a vertical channel for immiscible viscous fluids.

The rate of heat transfer in a vertical channel could be enhanced by using special inserts. Heat transfer in such partially divided enclosures has received attention previously due to its applications to design energy efficient buildings and reduction of heat loss from flat plate solar collectors. When the channel is divided into several passages by means of plane baffles, as usually occurs in heat exchangers or electronic equipment, it is quite possible to enhance the heat transfer performance between the walls and fluid by the adjustments of each baffle position and strengths of the separate flow streams. In such configurations, perfectly conductive and thin baffles may be used to avoid significant increase of the transverse thermal resistance. For a number of fluids, the density-temperature relation exhibits an extreme. Because the coefficient of thermal expansion changes signs at this extremum. Simple linear relations for density as a function of temperature are inadequate near the extremum. Dutta and Dutta [8] first reported the enhancement of heat transfer with inclined solid and perforated baffles. Later Dutta and Hossian [9] did the experimental study to analyze the local heat transfer characteristics in a rectangular channel with inclined solid and perforated baffles. Salah El-Din [10, 11] published a series of papers on mixed convection in a vertical channel by introducing a perfectly conducting baffle.

Mousavi and Hooman [12] studied numerically the fluid flow and heat transfer in the entrance region of a two dimensional horizontal channel with isothermal walls and with staggered baffles. Heat transfer enhancement in a heat exchanger tube by installing a baffle was reported by Nasiruddin and Siddiqui [13]. They found that the average Nusselt number for the two baffles case is 20% higher than the one baffle case and 82% higher than the no baffle case. Recently, Prathap Kumar et al. [14, 15] studied the flow characteristics of fully developed free convection flow of a Walters fluid (Model B') in a vertical channel divided into two passages. Umavathi [16] analyzed the effect of the presence of a thin perfectly conductive baffle on the fully developed laminar mixed convection in a vertical channel containing micropolar fluid.

Combining heat and mass transfer problems with a chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In such processes as drying, energy transfer in a wet cooling tower, and the flow in a desert cooler, heat and mass transfer occurs simultaneously. Mixed convection processes involving the combined mechanisms are also encountered in many natural processes, such as evaporation, condensation, and agricultural drying, and in many industrial applications, such as the curing of plastics and the manufacture of pulp-insulated cables [17]. In many chemical engineering processes, chemical reactions take place between a foreign mass and the working fluid which moves due to the stretch of a surface.

The order of the chemical reactions depends on several factors. One of the simplest chemical reactions is the first-order reaction in which the rate of the reaction is directly proportional to the species concentration. Chamkha [18] studied the analytical solutions for heat and mass transfer by the laminar flow of a Newtonian, viscous, electrically conducting and heat generating/absorbing fluid on a continuously moving vertical permeable surface in the presence of a magnetic field and the first-order chemical reaction. Muthucumaraswamy and Ganesan [19] studied the numerical solution for the transient natural convection flow of an incompressible viscous fluid past an impulsively started semi-infinite isothermal vertical plate with the mass diffusion, taking into account a homogeneous chemical reaction of the first order.

The coupled nonlinear ordinary differential equations governing the flow are solved using regular perturbation method which is the oldest method used by many researchers. In this paper a new method known as Differential Transform method is applied to find the analytical solution. The main advantage of DTM is that it can be applied directly to nonlinear differential equations without requiring linearization, discritization, or perturbation. This method is well addressed in [20-24]. Recently Umavathi et al. [25] solved the coupled nonlinear equations governing the flow for magnetoconvection in a vertical channel for open and short circuits usng Differential Transform method. The aim of this paper is to investigate effect of first order chemical reaction of viscous fluid in a vertical channel in the presence of a thin conducting baffle. After inserting the baffle, the fluid in stream-1 is concentrated. Analytical solutions are found using PM and using DTM.

II. MATHEMATICAL FORMULATION

Consider a steady, two-dimensional laminar fully developed free convection flow in an open ended vertical channel filled with purely viscous fluid. The X-axis is taken vertically upward, and parallel to the direction of buoyancy, and the Y-axis is normal to it. Walls are maintained at a constant temperature and the fluid properties are assumed to be constant. The channel is divided into two passages by means of thin, perfectly conducting plane baffle and each stream will have its own pressure gradient and hence the velocity will be individual in each stream.

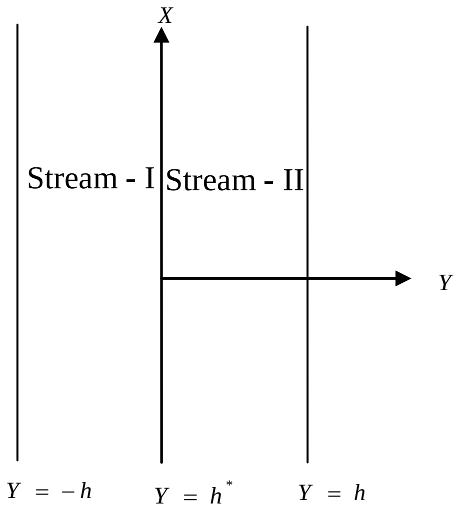

Figure 1. Physical configuration.

The governing equations for velocity, temperature and concentrations are

Stream-I

$$\rho g \beta_T \left(T_1 - T_{W_2}\right) + \rho g \beta_C (C_1 - C_0) - \frac{\partial P}{\partial X} + \mu \frac{d^2 U_1}{dY^2} = 0 \tag{1}$$

$$\frac{d^2 T_1}{dY^2} + \frac{\nu}{\alpha C_P}\left(\frac{dU_1}{dY}\right)^2 = 0 \tag{2}$$

$$D\frac{d^2 C}{dY^2} - kC = 0 \tag{3}$$

Stream-II

$$\rho g \beta_T \left(T_2 - T_{W_2}\right) - \frac{\partial P}{\partial X} + \mu \frac{d^2 U_2}{dY^2} = 0 \tag{4}$$

$$\frac{d^2 T_2}{dY^2} + \frac{\nu}{\alpha C_P}\left(\frac{dU_2}{dY}\right)^2 = 0 \tag{5}$$

subject to the boundary and interface conditions on velocity, temperature and concentration as

$U_1 = 0$, $T_1 = T_{W_1}$, $C = C_1$, at $Y = -h$

$U_2 = 0$, $T_2 = T_{W_2}$, at $Y = h$

$$U_1 = 0\ , U_2 = 0\ ,\ T_1 = T_2\ , \frac{dT_1}{dY} = \frac{dT_2}{dY}\ , C = C_2\ , \text{at } Y = h^* \tag{6}$$

Introducing the following non-dimensional variables,

$$u_i = \frac{U_i}{\bar{U}_1},\ \theta_i = \frac{T_i - T_{W_2}}{T_{W_1} - T_{W_2}},\ G_r = \frac{g\beta_T \Delta T h^3}{\upsilon^2},\ G_C = \frac{g\beta_C \Delta C h^3}{\upsilon^2},\ \phi = \frac{C_1 - C_0}{C_1 - C_0},\ \mathrm{Re} = \frac{\bar{U}_1 h}{\upsilon},\quad Br = \frac{\bar{U}_1^2 \mu}{k\Delta T},$$

$$p = \frac{h^2}{\mu \bar{U}_1}\frac{\partial p}{\partial X}\ ,\ \Delta T = T_{W_2} - T_{W_1},\ \Delta C = C_1 - C_0,\ Y^* = \frac{y^*}{h},\ Y = \frac{y}{h} \tag{7}$$

where $i = 1, 2$.

The momentum, energy and concentration equations corresponding to stream-I and stream-II become

Stream-I

$$\frac{d^2u_1}{dy^2} + GR_T\theta_1 + GR_c\phi - p = 0 \tag{8}$$

$$\frac{d^2\theta_1}{dy^2} + Br\left(\frac{du_1}{dy}\right)^2 = 0 \tag{9}$$

$$\frac{d^2\phi}{dy^2} - \alpha^2\phi = 0 \tag{10}$$

Stream-II

$$\frac{d^2u_2}{dy^2} + GR_T\theta_2 - p = 0 \tag{11}$$

$$\frac{d^2\theta_2}{dy^2} + Br\left(\frac{du_2}{dy}\right)^2 = 0 \tag{12}$$

subject to the boundary conditions,

$u_1 = 0$, $\theta_1 = 1$, $\phi = 1$, at $y = -1$

$u_2 = 0$, $\theta_2 = 0$, at $y = 1$

$$u_1 = 0,\ u_2 = 0,\ \theta_1 = \theta_2,\ \frac{d\theta_1}{dy} = \frac{d\theta_2}{dy},\ \phi = n, \text{at } y = y^* \tag{13}$$

where $\alpha = \frac{kh^2}{D}$, $n = \frac{C_2 - C_0}{C_1 - C_0}$.

III. SOLUTIONS

The exact solution for concentration distribution is found using Eq. (10) and is given by

$$\phi = B_1 Cosh(\alpha y) + B_2 Sinh(\alpha y) \tag{14}$$

3.1 Perturbation Method

Equations (8), (9), (11) and (12) are coupled non-linear ordinary differential equations. Approximate solutions can be found by using the regular perturbation method and Differential Transform method. The perturbation parameter is considered as Brinkman number Br. Adopting this method, solutions for velocity and temperature are assumed in the form

$$u_i(y) = u_{i0}(y) + Bru_{i1}(y) + Br^2u_{i2}(y) + \ldots \tag{15}$$

$$\theta_i(y) = \theta_{i0}(y) + Br\theta_{i1}(y) + Br^2\theta_{i2}(y) + \ldots \tag{16}$$

where the subscript $i = 1$ and 2 represents stream-I and stream-II respectively.

Substituting Eqs. (15) and **(**16) into Eqs. (8), (9), (11) and (12) and equating the coefficients of like power of Br to zero and one, we obtain the zeroth and first order equations as

Stream-I

Zeroth order equations

$$\frac{d^2\theta_{10}}{dy^2} = 0 \tag{17}$$

$$\frac{d^2u_{10}}{dy^2} + GR_T\theta_{10} + GR_c\phi - p = 0 \tag{18}$$

First order equations

$$\frac{d^2\theta_{11}}{dy^2} + \left(\frac{du_{10}}{dy}\right)^2 = 0 \tag{19}$$

$$\frac{d^2u_{11}}{dy^2} + GR_T\theta_{11} = 0 \tag{20}$$

Stream-II

Zeroth order equations

$$\frac{d^2\theta_{20}}{dy^2} = 0 \tag{21}$$

$$\frac{d^2u_{20}}{dy^2} + GR_T\theta_{20} - p = 0 \tag{22}$$

First order equations

$$\frac{d^2\theta_{21}}{dy^2} + \left(\frac{du_{20}}{dy}\right)^2 = 0 \tag{23}$$

$$\frac{d^2u_{21}}{dy^2} + GR_T\theta_{21} = 0 \tag{24}$$

The corresponding zeroth order boundary conditions reduces to

$u_{10} = 0,\ \theta_{10} = 1$, at $y = -1$

$u_{20} = 0,\ \theta_{20} = 0$, at $y = 1$

$$u_{10} = 0,\ u_{20} = 0,\ \theta_{10} = \theta_{20},\ \frac{d\theta_{10}}{dy} = \frac{d\theta_{20}}{dy}, \text{ at } y = y^* \tag{25}$$

The corresponding first order boundary conditions reduces to

$u_{11} = 0,\ \theta_{11} = 0$ at $y = -1$

$u_{21} = 0,\ \theta_{21} = 0$ at $y = 1$

$$u_{11} = 0,\ u_{21} = 0,\ \theta_{11} = \theta_{21},\ \frac{d\theta_{11}}{dy} = \frac{d\theta_{21}}{dy} \text{ at } y = y^* \tag{26}$$

The solutions of zeroth and first order equations (17) to (24) using the boundary conditions as given in Eqs. (25) and (26) are

Zeroth-order solutions

Stream-I

$$\theta_{10} = C_1 y + C_2 \tag{27}$$

$$u_{10} = A_2 + A_1 y + r_1 y^2 + r_2 y^3 + r_4 Cosh(\alpha y) + r_5 Sinh(\alpha y) \tag{28}$$

Stream-II

$$\theta_{20} = C_3 y + C_4 \tag{29}$$

$$u_{20} = A_4 + A_3 y + r_5 y^2 + r_6 y^3 \tag{30}$$

First order solutions

Stream-I

$$\begin{aligned}\theta_{11} &= G_2 + G_1 y + p_1 y^2 + p_2 y^3 + p_3 y^4 + p_4 y^5 + p_5 y^6 + p_6 Cosh(2\alpha y)\\ &+ p_7 Sinh(2\alpha y) + p_8 Cosh(\alpha y) + p_9 Sinh(\alpha y) + p_{10} y Cosh(\alpha y)\\ &+ p_{11} y Sinh(\alpha y) + p_{12} y^2 Cosh(\alpha y) + p_{13} y^2 Sinh(\alpha y)\end{aligned} \tag{31}$$

$$\begin{aligned}u_{11} &= G_6 + G_5 y + R_1 y^2 + R_2 y^3 + R_3 y^4 + R_4 y^5 + R_5 y^6 + R_6 y^7 + R_7 y^8\\ &+ R_8 Cosh(2\alpha y) + R_9 Sinh(2\alpha y) + R_{10} Cosh(\alpha y) + R_{11} Sinh(\alpha y)\\ &+ R_{12} y Cosh(\alpha y) + R_{13} y Sinh(\alpha y) + R_{14} y^2 Cosh(\alpha y) + R_{15} y^2 Sinh(\alpha y)\end{aligned} \tag{32}$$

Stream-II

$$\theta_{21} = G_4 + G_3 y + q_1 y^2 + q_2 y^3 + q_3 y^4 + q_4 y^5 + q_5 y^6 \tag{33}$$

$$u_{21} = G_8 + G_7 y + S_1 y^2 + S_2 y^3 + S_3 y^4 + S_4 y^5 + S_5 y^6 + S_6 y^7 + S_7 y^8 \tag{34}$$

3.2 Basic concepts of the differential transform method

The analytical solutions obtained in Section 3.1 are valid only for small values of Brinkman number Br . In many practical problems mentioned earlier, the values of Br are usually large. In that case analytical solutions are difficult, and hence we resort to semi-numerical-analytical method known as Differential Transform method (DTM). The general concept of DTM is explained here: The k^{th} differential transformation of an analytical function $F(k)$ is defined as (Zhou [20])

$$F(k) = \frac{1}{k!}\left[\frac{d^k f(\eta)}{d\eta^k}\right]_{\eta=\eta_0}, \tag{35}$$

and the inverse differential transformation is given by

$$f(\eta) = \sum_{k=0}^{\infty} F(k)(\eta-\eta_0)^k, \tag{36}$$

Combining Eqs. (35) and (36), we obtain

$$f(\eta) = \sum_{k=0}^{\infty} \frac{(\eta-\eta_0)^k}{k!} \left.\frac{d^k f(\eta)}{d\eta^k}\right|_{\eta=\eta_0}, \tag{37}$$

From Eqs. (35)–(37), it can be seen that the differential transformation method is derived from Taylor's series expansion. In real applications the sum $\sum_{k=n}^{\infty} F(k)(\eta-\eta_0)^k$ is very small and can be neglected when n is sufficiently large. So $f(\eta)$ can be expressed by a finite series, and Eqn. (36) may be written as

$$f(\eta) = \sum_{k=0}^{n} F(k)(\eta-\eta_0)^k, \tag{38}$$

where the value of n depends on the convergence requirement in real applications and $F(k)$ is the differential transform of $f(\eta)$. Table 1 lists the basic mathematics operations frequently used in the following analysis.

Table 1 The operations for the one-dimensional differential transform method.

Original function	Transformed function
$y(\eta) = g(\eta) \pm h(\eta)$	$Y(k) = G(k) \pm H(k)$
$y(\eta) = \alpha\, g(\eta)$	$Y(k) = \alpha\, G(k)$
$y(\eta) = \dfrac{dg(\eta)}{d\eta}$	$Y(k) = (k+1)G(k+1)$
$y(\eta) = \dfrac{d^2 g(\eta)}{d\eta^2}$	$Y(k) = (k+1)(k+2)G(k+2)$
$y(\eta) = g(\eta)h(\eta)$	$Y(k) = \sum_{l=0}^{k} G(l)H(k-l)$
$y(\eta) = \eta^m$	$Y(k) = \delta(k-m) = \begin{cases} 1, \text{ if } k = m \\ 0, \text{ if } k \neq m \end{cases}$

Taking differential transform of Eqs. (8), (9), (11) and (12), one can obtain the transformed equations as

Stream-I

$$\overline{U}_1(k+2) = -\frac{1}{(k+1)(k+2)}\left(GR_T\overline{\Theta}_1(k) + GR_c\overline{\Phi}(k) - p\,\delta(k)\right) \tag{39}$$

$$\overline{\Theta}_1(k+2) = -\frac{Br}{(k+1)(k+2)}\sum_{r=0}^{k}(k-r+1)(r+1)\overline{U}_1(k-r+1)\overline{U}_1(r+1) \tag{40}$$

$$\overline{\Phi}(k+2) = \frac{\alpha^2\overline{\Phi}(k)}{(k+1)(k+2)} \tag{41}$$

Stream-II

$$\overline{U}_2(k+2) = -\frac{1}{(k+1)(k+2)}\left(GR_T\overline{\Theta}_2(k) - p\,\delta(k)\right) \tag{42}$$

$$\overline{\Theta}_2(k+2) = -\frac{Br}{(k+1)(k+2)}\sum_{r=0}^{k}(k-r+1)(r+1)\overline{U}_2(k-r+1)\overline{U}_2(r+1) \tag{43}$$

where, $\overline{U}_1(k)$, $\overline{U}_2(k)$, $\overline{\Theta}_1(k)$, $\overline{\Theta}_2(k)$ and $\overline{\Phi}(k)$ are the transformed notations of $u_1(y)$, $u_2(y)$, $\theta_1(y)$, $\theta_2(y)$ and $\phi_1(y)$ respectively. $\delta(k) = \begin{cases} 1, \text{ if } k = 0 \\ 0, \text{ if } k > 0 \end{cases}$.

The following are the transformed initial conditions

$$\overline{U}_1(0) = c_1,\ \overline{U}_1(1) = c_2,\ \overline{U}_2(0) = c_3,\ \overline{U}_2(1) = c_4,$$
$$\overline{\Theta}_1(0) = d_1,\ \overline{\Theta}_1(1) = d_2,\ \overline{\Theta}_2(0) = d_3,\ \overline{\Theta}_2(1) = d_4,$$
$$\overline{\Phi}(0) = e_1,\ \overline{\Phi}(1) = e_2 \tag{44}$$

Using the boundary condition (13), we can evaluate c_1, c_2, c_3, c_4, d_1, d_2, d_3, d_4, e_1 and e_2.

IV. RESULTS AND DISCUSSIONS

The objective of the present study is to understand the characteristics of mixed convection of a viscous fluid in a vertical double passage channel in the presence of chemical reaction. The solutions are found using perturbation method and Differential Transformation method. The physical parameters such thermal Grashoff number GR_T, mass Grashoff number GR_C, Brinkman number Br (or perturbation parameter) and chemical reaction parameter α, are fixed as 5, 5, 0.1, and 0.5 respectively, for all the graphs except the varying one. The effect of these parameters on velocity, temperature and concentration are shown in Figs. 2 – 10. The effect of thermal Grashoff number GR_T (ratio of Grashoff number to Reynolds number) on the velocity and temperature is shown in Figs. 2a,b,c and Figs. 3a,b,c at all three different baffle positions (i.e. $y^* = -0.8$, 0.0 and 0.8). As the thermal Grashoff number increases, the velocity and temperature increases at all the baffle position whereas the maximum velocity field is observed in the wider stream. It is also observed form Figs. 3a,b,c that the temperature distribution is more effective near the left wall when compared to right wall. Further it is well-known that since Grashoff number is the ratio of buoyancy force to viscous force, increase in Grashoff number is to increase the buoyancy force and hence increases the concentration also. Therefore as the thermal Grashoff number increases velocity and temperature increases at all baffle position in both the streams. The effect of mass Gerashof number GR_C (ratio of modified Grashoff number to Reynolds number) is shown in Figs. 4a,b,c for velocity field and in Figs. 5a,b,c for the temperature field. Here also the effect of GR_C is to increase the velocity and temperature field in both the streams. It is seen from Figs. 4a and 5a ($y^* = -0.8$) that the effect of GR_C on the velocity and temperature fields is not effective whereas when the baffle position is at $y^* = 0.0$ and 0.8 the flow field is enhanced as GR_C increases. The similar result is also observed by Fasogbon [26] for irregular channel.

The effect of Brinkman number Br on the velocity and temperature fields are shown in Figs. 6a,b,c and Figs. 7a,b,c respectively. As the Brinkman number increases, both the velocity and temperature increases in both the streams at all baffle positions. One can see from temperature equation that increase in Brinkman number increases the viscous dissipation and hence the temperature increases, which intern influences the velocity and temperature. The effect of first order chemical reaction parameter α, on the velocity, temperature and concentration fields is shown in Figs. 8a,b,c, Figs. 9a,b,c and Figs. 10a,b,c respectively. As α increases the velocity and temperature decreases in stream-I, and remains invariant in stream-II when the baffle position $y^* = -0.8$. But when the baffle position is at $y^* = 0$ & 0.8 the effect of α is more effective in stream –I and less effective in stream –II. This is because the fluid is concentrated in stream-I only. The effect of chemical reaction parameter α is to decrease the concentration distribution as seen in Figs. 10a,b,c, which is the similar result obtained by Srinivas and Muturajan [27] for mixed convective flow in a vertical channel. It is observed from Tables 2a, 3a and 4a that results of DTM and PM agree well in the absence of Brinkman number at all the baffle positions. For large values of Brinkman number $(Br \neq 0)$, DTM and PM solutions show difference as seen in Tables 2(b,c) to 4(b,c). It is also observed from these tables that the error of DTM and PM is very less in smaller stream when compared to bigger stream at all baffle position for $Br \neq 0$.

V. CONCLUSION

The effect of first order chemical reaction in a vertical double passage channel filled with purely viscous fluid was investigated. The solutions of the governing equations and the associated boundary conditions have been obtained by using regular perturbation method and differential transform method. Main findings are summarized as follows:

[1] Increasing thermal Grashoff number, mass Grashoff number and Brinkman number increases the velocity and temperature in both the streams at all different baffle position.
[2] Increase in the chemical reaction parameter suppresses the velocity and temperate in stream-I and remains invariant in stream-II.
[3] The use of baffle in the flow channel resulted in the heat transfer enhancement as high as compared to the heat transfer in a channel without baffle.
[4] Chemical reaction parameter was to decrease the flow field.
[5] An excellent agreement was observed with the results of DTM and PM for small values of Brinkman number.

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NOMENCLATURE

Br Brinkman number $\left(\dfrac{\overline{u_1^2}\,\mu_1}{K\,\Delta T}\right)$

C_1 the Concentration in Stream-I

C_0 reference concentration

C_p specific heat at constant pressure

c_p dimensionless specific heat at constant pressure

D diffusion coefficients

g acceleration due to gravity

Gr Grashoff number $\left(\frac{h^3 g\beta\Delta T}{\nu^2}\right)$

G_c modified Grashoff Number $\left(\frac{g\ \beta c\ \Delta C\ h^3}{\gamma^2}\right)$

GR_T thermal Grashoff number $(= Gr\ /\ \mathrm{Re}\ )$

GR_C mass Grashof number $(= Gc\ /\ \mathrm{Re}\ )$

h channel width

h^* width of passage

k thermal conductivity of fluid

p non-dimensional Pressure Gradient $(\frac{h^2}{\overline{U}_1\mu}\frac{\partial p}{\partial X})$

Re Reynolds number $(\frac{\overline{U}_1 h}{\gamma})$

$T_1,\ T_2$ dimensional temperature distributions

T_{w_1}, T_{w_2} temperatures of the boundaries

$\overline{U}_1$ reference velocity

U_1, U_2 dimensional velocity distributions

u_1, u_2 non dimensional Velocities in Stream-I, Stream-II

y^* baffle position

GREEK SYMBOLS

α chemical reaction parameters

β_T coefficients of thermal expansion

β_C coefficients of concentration expansion

$\Delta T, \Delta C$ difference in Temperatures & Concentration

ε perturbation Parameter

θ_i non-dimensional temperature $\left(\frac{T_i - T_{W_2}}{T_{W_1} - T_{W_2}}\right)$

γ kinematics viscosity

ϕ non-dimensional concentrations

ρ density

μ viscosity

SUBSCRIPTS

i refer quantities for the fluids in stream-I and stream-II, respectively.

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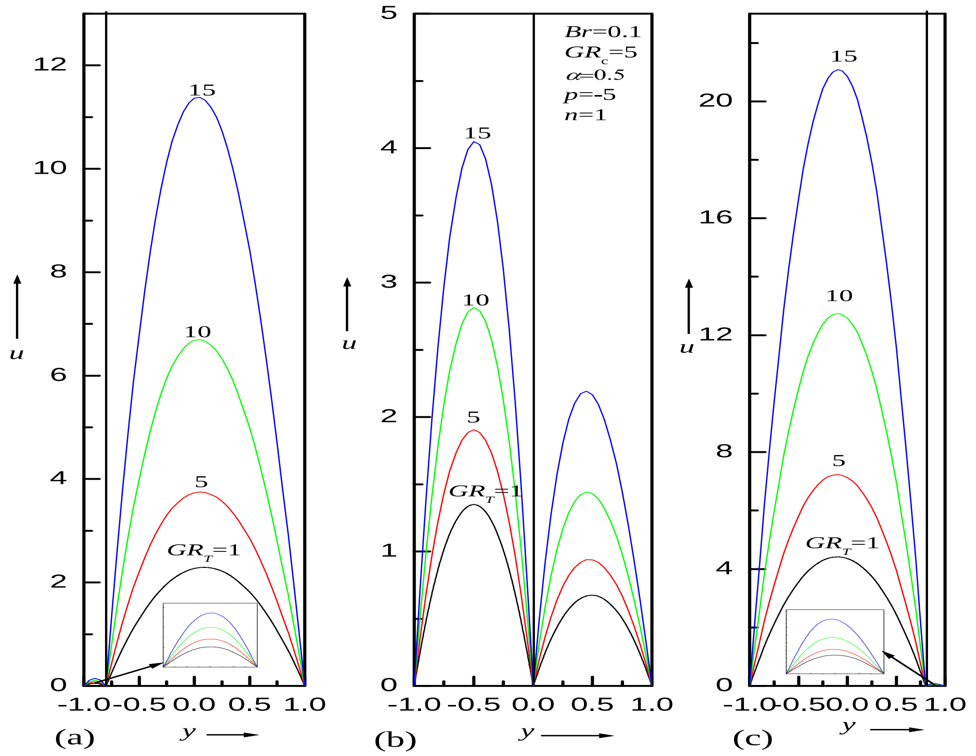
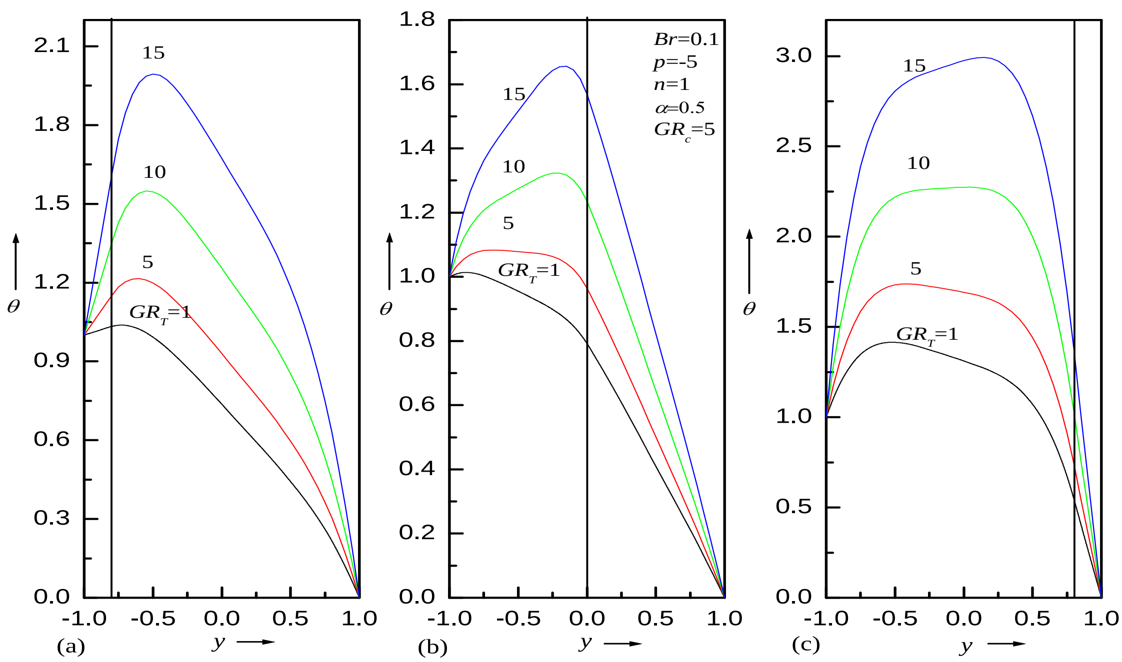


Fig.2: velocity distribution for different values of thermal Grashof number GR_T at (a) y*=-0.8 (b)y*=0.0 (c) y*=0.8



Figu.3: Temperature profile for different values of ratio of Grashof number to Reynolds number GR_T at (a)y*=0.8 (b)y*=0 (c)y*=0.8

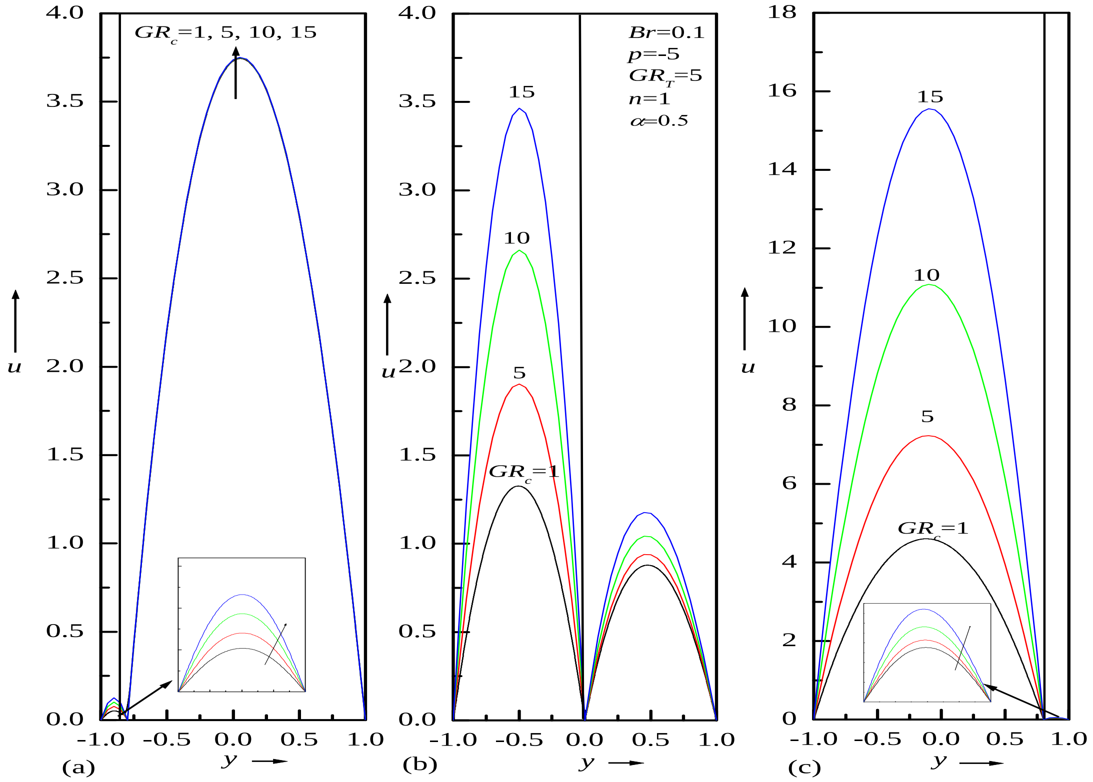


Fig.4: Velocity profile for different values ofratio of modified Grashoff number to Reynolds number GR_C at (a)y*=-0.8 (b)y*=0 (c)y*=0.8

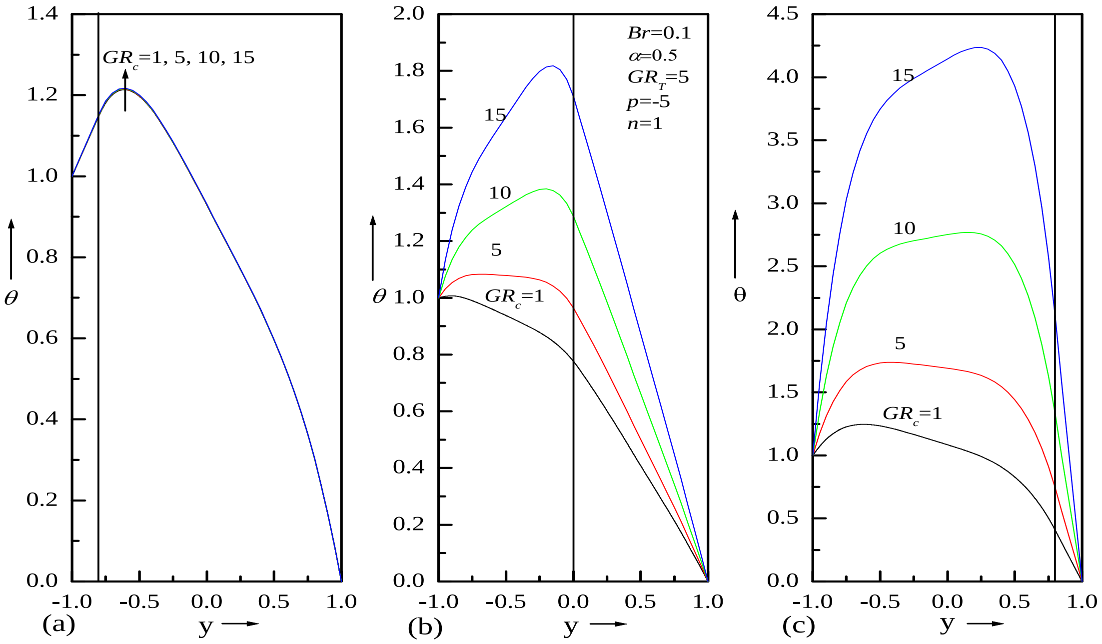


Fig.5: Temperature profile for different values of ratio of modified Grashof number to Reynolds number GR_C at (a)y*=-0.8 (b)y*=0 (c)y*=0.8

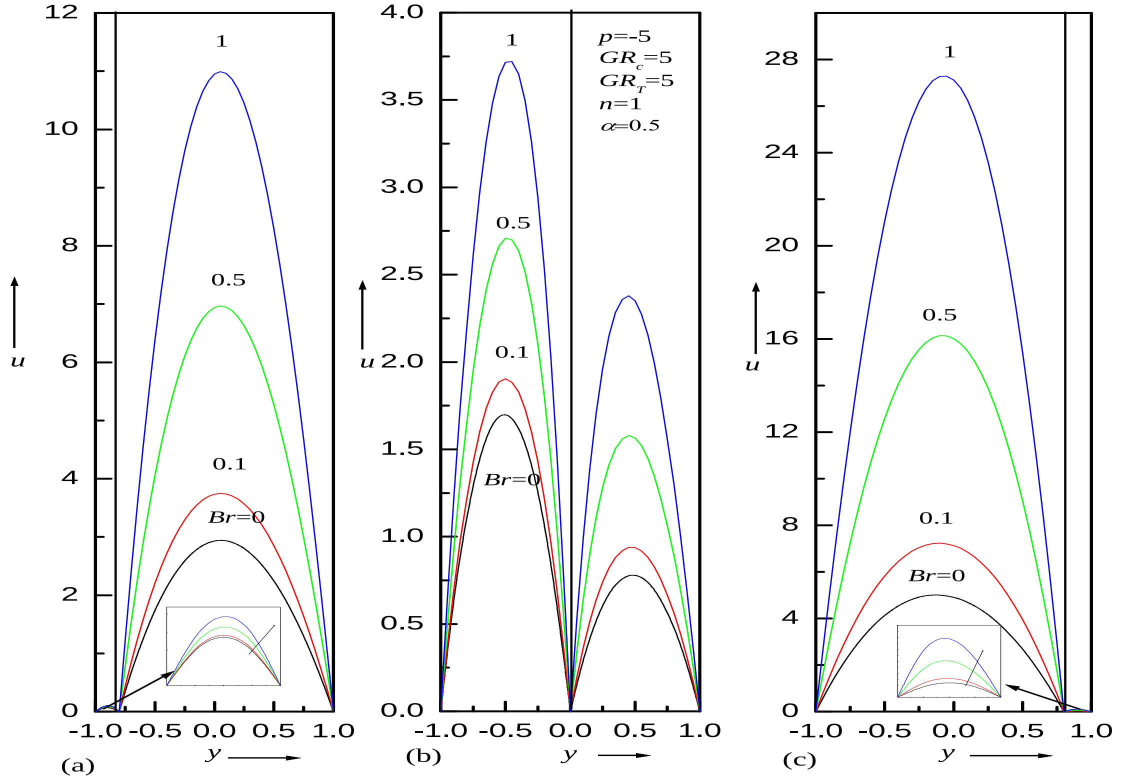


Fig.6: Velocity for different values of Brinkman number Br
(a)y*=-0.8 (b)y*=0 (c)y*=0.8

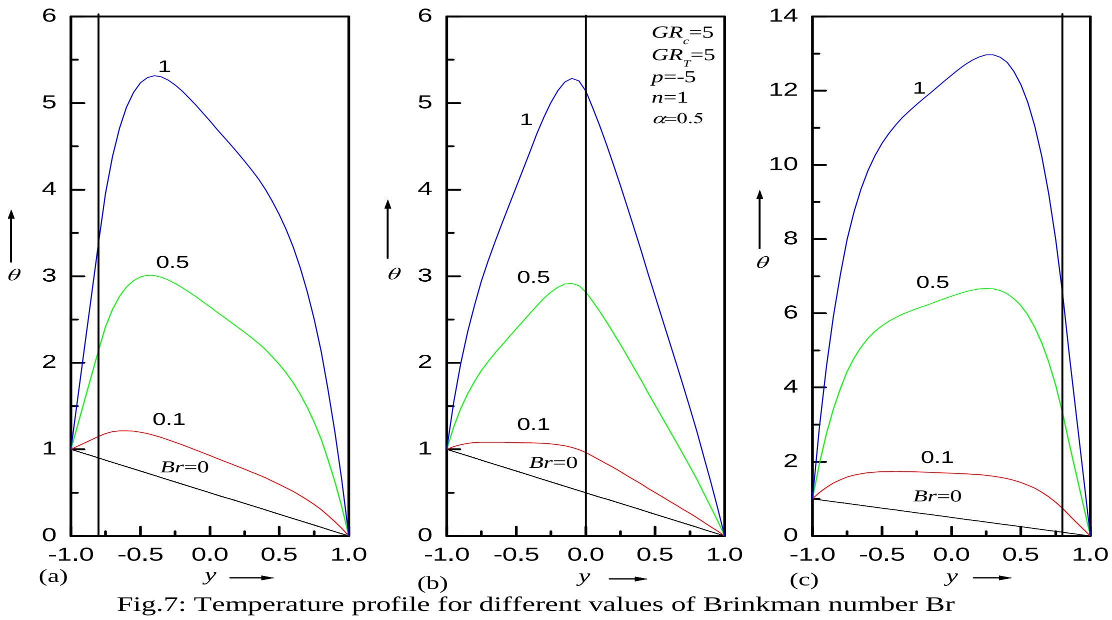


Fig.7: Temperature profile for different values of Brinkman number Br
at (a) y*=-0.8 (b)y*=0 (c) y*=0.8

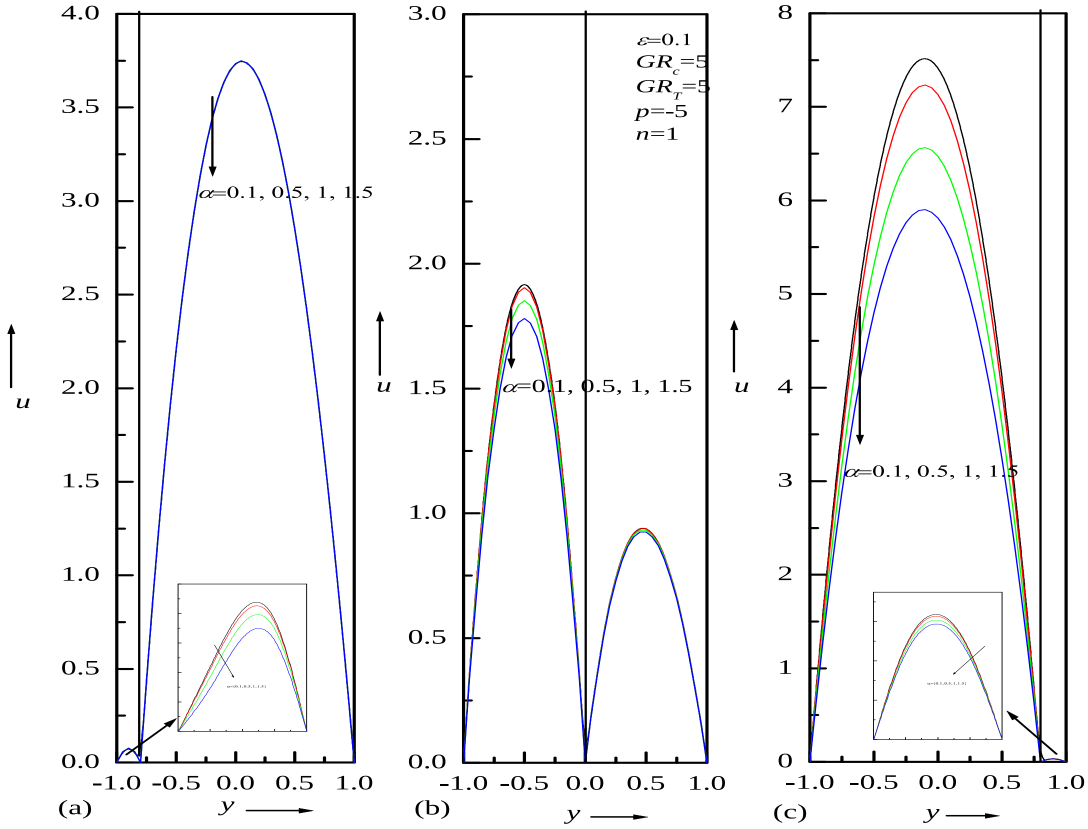


Fig.8: Velocity profile for different values of chemical reaction parameter α at (a) y*=-0.8 (b)y*=0 (c) y*=0.8

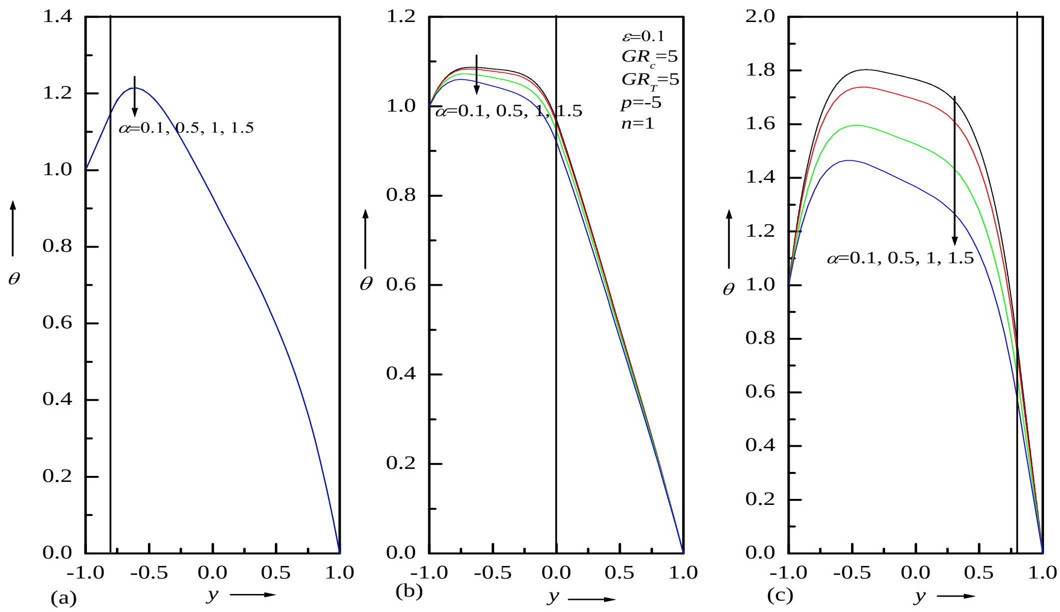


Fig.9: Temperature profile for different values of chemical reaction parameter α at (a) y*=-0.8 (b)y*=0 (c) y*=0.8

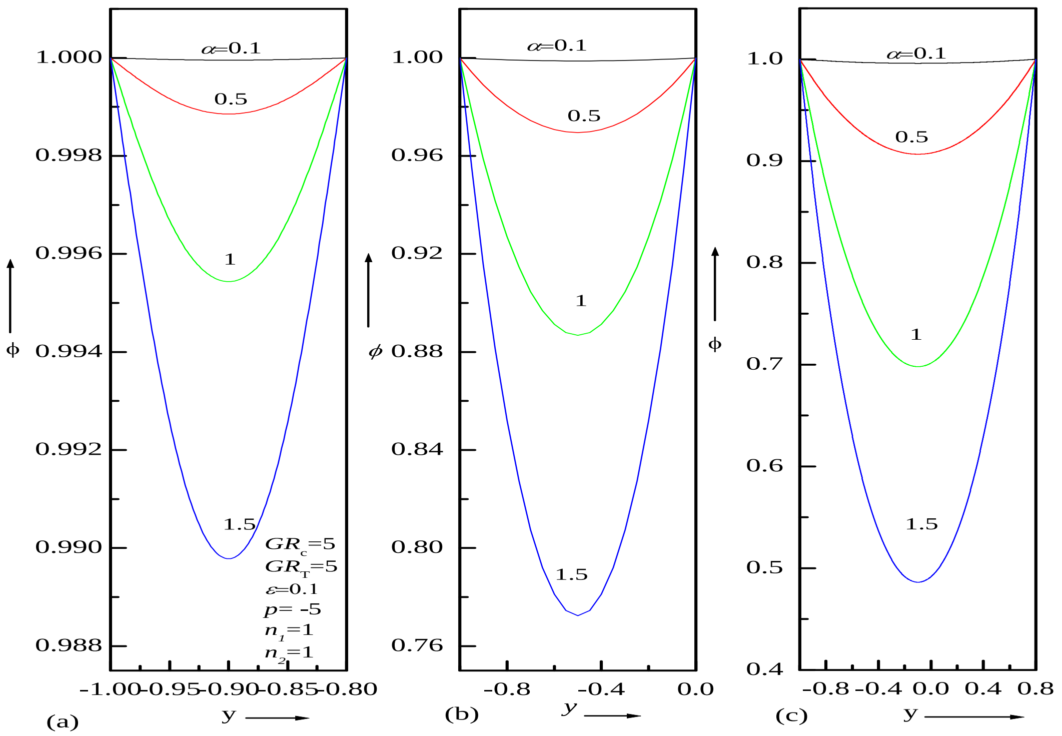


Figure10.Concentration profile for different values of chemical reaction parameter α

Table 2a Comparison of velocity and temperature with $Br = 0$, $GR_T = 5$, $GR_C = 5$, $p = -5$ and $y^* = 0.0$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.0000	1.000000	1.000000	0.0000
-0.75	1.266461	1.266461	0.0000	0.875000	0.875000	0.0000
-0.5	1.659656	1.659656	0.0000	0.750000	0.750000	0.0000
-0.25	1.227398	1.227398	0.0000	0.625000	0.625000	0.0000
0	0	0	0.0000	0.500000	0.500000	0.0000
0.25	0.605469	0.605469	0.0000	0.375000	0.375000	0.0000
0.5	0.781250	0.781250	0.0000	0.250000	0.250000	0.0000
0.75	0.566406	0.566406	0.0000	0.125000	0.125000	0.0000
1	0	0	0.0000	0	0	0.0000

Table 2b Comparison of velocity and temperature with $Br = 0.05$, $GR_T = 5$, $GR_C = 5$, $p = -5$ and $y^* = 0.0$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.0000	1.000000	1.000000	0.0000
-0.75	1.339968	1.329565	0.0104	0.989529	0.973754	0.0158
-0.5	1.771965	1.755951	0.0160	0.933166	0.907116	0.0261
-0.25	1.321337	1.307845	0.0135	0.870308	0.834778	0.0355
0	0	0	0.0000	0.761836	0.722594	0.0392
0.25	0.682521	0.670711	0.0118	0.583393	0.551573	0.0318
0.5	0.870491	0.856765	0.0137	0.393647	0.371510	0.0221
0.75	0.622964	0.614236	0.0087	0.202259	0.190149	0.0121
1	0	0	0.0000	0	0	0.0000

Table 2c Comparison of velocity and temperature with $Br = 0.15$, $GR_T = 5$, $GR_C = 5$, $p = -5$ and $y^* = 0.0$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.0000	1.000000	1.000000	0.0000
-0.75	1.651154	1.455775	0.1954	1.465379	1.171263	0.2941
-0.5	2.249870	1.948541	0.3013	1.711319	1.221349	0.4900
-0.25	1.723060	1.468738	0.2543	1.925328	1.254333	0.6710
0	0	0	0.0000	1.915429	1.167783	0.7476
0.25	1.027983	0.801196	0.2268	1.514892	0.904720	0.6102
0.5	1.271710	1.007795	0.2639	1.040516	0.614531	0.4260
0.75	0.877917	0.709895	0.1680	0.554797	0.320447	0.2344
1	0	0	0.0000	0	0	0.0000

Table 3a Comparison of velocity and temperature with $Br = 0$, $GR_T = 5$, $GR_C = 5$, $p = -5$ and $y^* = -0.8$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.0000	1.000000	1.000000	0.0000
-0.95	0.055395	0.055395	0.0000	0.975000	0.975000	0.0000
-0.9	0.073646	0.073646	0.0000	0.950000	0.950000	0.0000
-0.85	0.055082	0.055082	0.0000	0.925000	0.925000	0.0000
-0.8	0	0	0.0000	0.900000	0.900000	0.0000
-0.5	1.743750	1.743750	0.0000	0.750000	0.750000	0.0000
-0.2	2.700000	2.700000	0.0000	0.600000	0.600000	0.0000
0.1	2.936250	2.936250	0.0000	0.450000	0.450000	0.0000
0.4	2.520000	2.520000	0.0000	0.300000	0.300000	0.0000
0.7	1.518750	1.518750	0.0000	0.150000	0.150000	0.0000
1	0	0	0.0000	0	0	0.0000

Table 3b Comparison of velocity and temperature with $Br = 0.05$, $GR_T = 5$, $GR_C = 5$, $p = -5$ and $y^* = -0.8$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.0000	1.000000	1.000000	0.0000
-0.95	0.056795	0.056380	0.0004	1.019848	1.006548	0.0133
-0.9	0.075886	0.075222	0.0007	1.039612	1.013017	0.0266
-0.85	0.057042	0.056460	0.0006	1.059364	1.019475	0.0399
-0.8	0	0	0.0000	1.079032	1.025854	0.0532
-0.5	2.076642	1.976262	0.1004	1.070430	0.974183	0.0962
-0.2	3.226545	3.067590	0.1590	0.925078	0.826975	0.0981
0.1	3.511194	3.337492	0.1737	0.748205	0.658004	0.0902
0.4	3.009255	2.861363	0.1479	0.565508	0.485024	0.0805
0.7	1.804303	1.717982	0.0863	0.341631	0.283674	0.0580
1	0	0	0.0000	0	0	0.0000

Table 3c Comparison of velocity and temperature with $Br = 0.09$, $GR_T = 5$, $GR_C = 5$, $p = -5$ and $y^* = -0.8$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.0000	1.000000	1.000000	0.0000
-0.95	0.061213	0.057167	0.0040	1.160698	1.031787	0.1289
-0.9	0.082935	0.076482	0.0065	1.320672	1.063431	0.2572
-0.85	0.063199	0.057563	0.0056	1.480419	1.095055	0.3854
-0.8	0	0	0.0000	1.639913	1.126537	0.5134
-0.5	3.134164	2.162271	0.9719	2.084440	1.153530	0.9309
-0.2	4.901119	3.361663	1.5395	1.958644	1.008556	0.9501
0.1	5.341092	3.658485	1.6826	1.698429	0.824406	0.8740
0.4	4.567218	3.134453	1.4328	1.413275	0.633044	0.7802
0.7	2.713644	1.877367	0.8363	0.952144	0.390614	0.5615
1	0	0	0.0000	0	0	0.0000

Table 4a Comparison of velocity and temperature with $Br = 0$, $GR_T = 5$, $GR_C = 5$, $p = -5$ and $y^* = 0.8$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.0000	1.000000	0.850000	0.0000
-0.7	2.720194	2.720194	0.0000	0.700000	0.550000	0.0000
-0.4	4.232842	4.232842	0.0000	0.400000	0.250000	0.0000
-0.1	4.649777	4.649777	0.0000	0.100000	0.100000	0.0000
0.2	4.052842	4.052842	0.0000	0.075000	0.050000	0.0000
0.5	2.495194	2.495194	0.0000	0.025000	0	0.0000
0.8	0	0	0.0000	1.000000	0.850000	0.0000
0.85	0.019844	0.019844	0.0000	0.400000	0.250000	0.0000
0.9	0.026250	0.026250	0.0000	0.100000	0.100000	0.0000
0.95	0.019531	0.019531	0.0000	0.075000	0.050000	0.0000
1	0	0	0.0000	0.025000	0	0.0000

Table 4b Comparison of velocity and temperature with $Br = 0.01$, $GR_T = 5$, $GR_C = 5$, $p = -5$ and $y^* = 0.8$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.0000	1.000000	1.000000	0.0000
-0.7	2.825637	2.816608	0.0090	0.924245	0.918020	0.0062
-0.4	4.412157	4.396750	0.0154	0.797661	0.789269	0.0084
-0.1	4.859428	4.841378	0.0181	0.658724	0.649363	0.0094
0.2	4.243934	4.227453	0.0165	0.517827	0.507653	0.0102
0.5	2.615207	2.604832	0.0104	0.362559	0.352732	0.0098
0.8	0	0	0.0000	0.160579	0.155260	0.0053
0.8	0	0	0.0000	0.160579	0.155260	0.0053
0.85	0.020506	0.020448	0.0001	0.120437	0.116447	0.0040
0.9	0.027007	0.026941	0.0001	0.080292	0.077632	0.0027
0.95	0.020005	0.019963	0.0000	0.040147	0.038817	0.0013
1	0	0	0.0000	0	0	0.0000

Table 4c Comparison of velocity and temperature with $Br = 0.05$, $GR_T = 5$, $GR_C = 5$, $p = -5$ and $y^* = 0.8$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.0000	1.000000	1.000000	0.0000
-0.7	3.707230	3.202265	0.5050	1.536854	1.190102	0.3468
-0.4	5.914596	5.052384	0.8622	1.616169	1.146343	0.4698
-0.1	6.618204	5.607785	1.0104	1.571084	1.046814	0.5243
0.2	5.848729	4.925897	0.9228	1.507863	0.938267	0.5696
0.5	3.624481	3.043386	0.5811	1.314728	0.763661	0.5511
0.8	0	0	0.0000	0.674938	0.376298	0.2986
0.8	0	0	0.0000	0.674938	0.376298	0.2986
0.85	0.026132	0.022866	0.0033	0.506220	0.282234	0.2240
0.9	0.033437	0.029704	0.0037	0.337487	0.188161	0.1493
0.95	0.024023	0.021690	0.0023	0.168751	0.094085	0.0747
1	0	0	0.0000	0	0	0.0000

Appendix

$$C_1 = -\frac{1}{2},\quad C_2 = \frac{1}{2}\quad ,C_3 = -\frac{1}{2},\quad C_4 = \frac{1}{2},\quad B_1 = \frac{Sinh(\alpha y^*) + n\,Sinh(\alpha)}{Sinh(\alpha y^*)Cosh(\alpha) + Sinh(\alpha)Cosh(\alpha y^*)},$$

$$B_2 = \frac{n\,Cosh(\alpha) - Cosh(\alpha y^*)}{Sinh(\alpha y^*)Cosh(\alpha) + Sinh(\alpha)Cosh(\alpha y^*)},\qquad r_1 = \frac{(p - GR_T C_2)}{2},\qquad r_2 = -\frac{GR_T C_1}{6},$$

$$r_3 = -\frac{GR_c B_1}{\alpha^2},\qquad r_4 = -\frac{GR_c B_2}{\alpha^2},\qquad r_5 = \frac{(p - GR_T C_4)}{2},\qquad r_6 = -\frac{GR_T C_3}{6},$$

$$A_1 = -\frac{\left(r_1\left(y^{*2} - 1\right) + r_2\left(y^{*3} + 1\right) + r_3\left(Cosh(\alpha y^*) - Cosh(\alpha)\right) + r_4\left(Sinh(\alpha y^*) + Sinh(\alpha)\right)\right)}{1 + y^*},$$

$$A_2 = A_1 - r_1 + r_2 - r_3 Cosh(\alpha) + r_4 Sinh(\alpha),\ A_3 = \frac{r_5\left(1 - y^{*2}\right) + r_6\left(1 - y^{*3}\right)}{(y^* - 1)},\ A_4 = -A_3 - r_5 - r_6$$

$$p_1 = -\frac{\left(2A_1^2 + r_4^2\alpha^2 - r_3^2\alpha^2\right)}{4},\ p_2 = -\frac{2A_1 r_1}{3},\ p_3 = -\frac{\left(4r_1^2 + 6A_1 r_2\right)}{12},\ p_4 = -\frac{3r_1 r_2}{5},\ p_5 = -\frac{3r_2^2}{10},$$

$$p_6 = -\frac{\left(r_3^2 + r_4^2\right)}{8},\ p_7 = -\frac{r_3 r_4}{4},\ p_8 = -\frac{\left(2A_1 r_4\alpha^2 - 8r_1 r_3\alpha + 36 r_2 r_4\right)}{\alpha^3},$$

$$p_9 = -\frac{\left(2A_1 r_3\alpha^2 - 8r_1 r_4\alpha + 36 r_2 r_3\right)}{\alpha^3},\ p_{10} = -\frac{\left(4r_1 r_4\alpha - 24 r_2 r_3\right)}{\alpha^2},\ p_{11} = -\frac{\left(4r_1 r_3\alpha - 24 r_2 r_4\right)}{\alpha^2},$$

$$p_{12} = -\frac{6r_2 r_4}{\alpha},\ p_{13} = -\frac{6r_2 r_3}{\alpha},\ q_1 = -\frac{A_3^2}{2},\ q_2 = -\frac{2A_3 r_5}{3},\ q_3 = -\frac{\left(2r_5^2 + 3A_3 r_6\right)}{6},\ q_4 = -\frac{3r_5 r_6}{5},$$

$$q_5 = -\frac{3r_6^2}{10},\quad T_1 = -\begin{pmatrix} p_1 - p_2 + p_3 - p_4 + p_5 + p_6 Cosh(2\alpha) - p_7 Sinh(2\alpha) + p_8 Cosh(\alpha) \\ - p_9 Sinh(\alpha) - p_{10} Cosh(\alpha) + p_{11} Sinh(\alpha) + p_{12} Cosh(\alpha) - p_{13} Sinh(\alpha) \end{pmatrix},$$

$$T_2 = -(q_1 + q_2 + q_3 + q_4 + q_5),$$

$$\begin{aligned} T_3 = {} & q_1 y^{*2} + q_2 y^{*3} + q_3 y^{*4} + q_4 y^{*5} + q_5 y^{*6} - p_1 y^{*2} - p_2 y^{*3} - p_3 y^{*4} - p_4 y^{*5} - p_5 y^{*6} \\ & - p_6 Cosh(2\alpha y^*) - p_7 Sinh(2\alpha y^*) - p_8 Cosh(\alpha y^*) - p_9 Sinh(\alpha y^*) \\ & - p_{10} y^* Cosh(\alpha y^*) - p_{11} y^* Sinh(\alpha y^*) - p_{12} y^{*2} Cosh(\alpha y^*) - p_{13} y^{*2} Sinh(\alpha y^*) \end{aligned},$$

$$\begin{aligned} T_4 = {} & 2q_1 y^* + 3q_2 y^{*2} + 4q_3 y^{*3} + 5q_4 y^{*4} + 6q_5 y^{*5} - 2p_1 y^* - 3p_2 y^{*2} - 4p_3 y^{*3} - 5p_4 y^{*4} - 6p_5 y^{*5} \\ & - 2\alpha p_6 Sinh(2\alpha y^*) - 2\alpha p_7 Cosh(2\alpha y^*) - p_8\alpha Sinh(\alpha y^*) - \alpha p_9 Cosh(\alpha y^*) \\ & - p_{10}\left(y^*\alpha Sinh(\alpha y^*) + Cosh(\alpha y^*)\right) - p_{11}\left(y^*\alpha Cosh(\alpha y^*) + Sinh(\alpha y^*)\right) \\ & - p_{12}\left(2y^* Cosh(\alpha y^*) + \alpha y^{*2} Sinh(\alpha y^*)\right) - p_{13}\left(2y^* Sinh(\alpha y^*) + \alpha y^{*2} Cosh(\alpha y^*)\right) \end{aligned},$$

$$G_1 = -\frac{\left(y^* T_4 + T_1 - T_2 - T_3 - T_4\right)}{2},\ G_2 = \frac{\left(T_1 + T_2 + T_3 + T_4\left(1 - y^*\right)\right)}{2},$$

$$G_3 = \frac{\left(-T_1 + T_2 + T_3 - T_4\left(1 + y^*\right)\right)}{2},\qquad G_4 = T_2 - G_3,\qquad R_1 = -\frac{GR_T G_2}{2},\qquad R_2 = -\frac{GR_T G_1}{6},$$

$$R_3 = -\frac{GR_T p_1}{12},\quad R_4 = -\frac{GR_T p_2}{20},\quad R_5 = -\frac{GR_T p_3}{30},\quad R_6 = -\frac{GR_T p_4}{42},\quad R_7 = -\frac{GR_T p_5}{56},$$

$$R_8 = -\frac{GR_T p_6}{4\alpha^2},\qquad R_9 = -\frac{GR_T p_7}{4\alpha^2},\qquad R_{10} = -\frac{\left(p_8\alpha^2 - 2p_{11}\alpha + 6p_{12}\right)GR_T}{\alpha^4},$$

$$R_{11} = -\frac{\left(p_9\alpha^2 - 2p_{10}\alpha + 6p_{13}\right)GR_T}{\alpha^4}, \quad R_{12} = -\frac{\left(p_{10}\alpha - 4p_{13}\right)GR_T}{\alpha^3}, \quad R_{13} = -\frac{\left(p_{11}\alpha - 4p_{12}\right)GR_T}{\alpha^3},$$

$$R_{14} = -\frac{GR_T\, p_{12}}{\alpha^2}, \quad R_{15} = -\frac{GR_T\, p_{13}}{\alpha^2}, \quad S_1 = -\frac{GR_T\, G_4}{2}, \quad S_2 = -\frac{GR_T\, G_3}{6}, \quad S_3 = -\frac{GR_T\, q_1}{12},$$

$$S_4 = -\frac{GR_T\, q_2}{20}, \quad S_5 = -\frac{GR_T\, q_3}{30}, \quad S_6 = -\frac{GR_T\, q_4}{42}, \quad S_7 = -\frac{GR_T\, q_5}{56},$$

$$T_5 = -\begin{pmatrix} R_1 - R_2 + R_3 - R_4 + R_5 - R_6 + R_7 + R_8\, Cosh(2\alpha) - R_9\, Sinh(2\alpha) + R_{10}\, Cosh(\alpha) \\ -R_{11}\, Sinh(\alpha) - R_{12}\, Cosh(\alpha) + R_{13}\, Sinh(\alpha) + R_{14}\, Cosh(\alpha) - R_{15}\, Sinh(\alpha) \end{pmatrix}$$

$$T_7 = \begin{pmatrix} R_1 y^{*2} + R_2 y^{*3} + R_3 y^{*4} + R_4 y^{*5} + R_5 y^{*6} + R_6 y^{*7} + R_7 y^{*8} + R_8\, Cosh(2\alpha y^*) + R_9\, Sinh(2\alpha y^*) \\ + R_{10}\, Cosh(\alpha y^*) + R_{11}\, Sinh(\alpha y^*) + R_{12}\, y^* Cosh(\alpha y^*) + R_{13} y^* Sinh(\alpha y^*) \\ + R_{14} y^{*2}\, Cosh(\alpha y^*) + R_{15} y^{*2}\, Sinh(\alpha y^*) \end{pmatrix}$$

$$G_5 = \frac{T_7 - T_5}{1 + y^*}, \; G_7 = \frac{T_6 - T_8}{1 - y^*}, \; G_6 = T_5 + G_5, \; G_8 = T_6 - G_7.$$